Limiting effects of geometrical and optical nonlinearities on the squeezing in optomechanics

P. Djorwé[a,**], S.G. Nana Engo[b,*], J.H. Talla Mbé[a], P. Woafo[a]

[a]*Laboratory of Modelling and Simulation in Engineering, Biomimetics and Prototypes, Faculty of Science, University of Yaoundé I, Cameroon*
[b]*Laboratory of Photonics, Faculty of Science, University of Ngaoundéré, Cameroon*

Abstract

In recent experiments, the re-thermalization time of the mechanical resonator is stated as the limiting factor for quantum applications of optomechanical systems. To explain the origin of this limitation, an analytical nonlinear investigation supported by the recent successful experimental laser cooling parameters is carried out in this work. To this end, the effects of geometrical and the optical nonlinearities on the squeezing are studied and are in a good agreement with the experimental results. It appears that highly squeezed state are generated where these nonlinearities are minimized and that high nonlinearities are limiting factors to reach the quantum ground state.

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1. Introduction

Squeezing is a beautiful quantum phenomenon with amazing potential applications [1, 2] of which the most recent are connected with continuous variables quantum information [3, 4] and ultrasensitive measurement of weak perturbations as the gravitational waves [5, 6, 7]. Squeezed states are nonclassical states in which the variance of at least one of the canonical variables is reduced below the noise level of zero point fluctuations. To generate squeezed states, the common technique consists to use an optical cavity filled with a nonlinear Kerr medium which is fed with an external pumping field [8]. With the recent advances in cooling techniques for nano scale optomechanical systems, various setups have been designed for quantum ground state engineering of mechanical mirrors with highly squeezed states of light [9, 10, 11, 12, 13, 14]. Indeed, with such technique it is now possible to obtain effective phonon number less than 1 [15, 16, 17, 18, 19, 20]. The limiting factors to obtain much lower phonon

*Corresponding author
**Principal corresponding author
Email address: snana@univ-ndere.cm (S.G. Nana Engo)

number are the re-thermalization time of the mechanical resonator $\tau_{th} = \frac{\hbar Q_m}{k_B T}$ (where $\hbar$ is Planck's constant, Q_m is the mechanical quality factor, k_B the Boltzmann constant and T the temperature of the support), which competes with the cooling, and the ubiquitous phase noise of the input laser which can create a discrepancy between experimental results and theoretical prediction [15, 16]. Nevertheless, a lot of theoretical studies on the subject has been carried out in the last decade and several proposals have been produced [21, 22, 23]. In Ref. [23] we applied the technique of back-action cooling to show that the cooling of the nanomechanical oscillator to its ground state is limited by the effects of both optical and mechanical nonlinearities.

In this paper, by using the parameters of the experimental laser cooling of Ref. [15], we extend the previous treatment to show through analytical study that there are the nonlinearities which limit the squeezing in optomechanics. The first one which depends on the geometry of the mechanical structure is known as the geometrical nonlinearity derives from the nonlinear dynamics of the beams [24, 25, 26]. The second one is the optical nonlinearity which appears as a nonlinear phase shift [10]. The geometrical nonlinearity which is always present and not negligible in nano resonators, is shown to be a limiting factor to reach the quantum ground state as suggested in Ref. [26]. In the same way, it is shown that high absolute values of the optical nonlinearity limit the squeezing of the output intensity.

The paper is structured as follows. Section 2 will set the stage for exploring the dynamics of our system, deriving in particular the nonlinear Quantum Langevin Equations and the linearized equations of motion. Sections 3 and 4 subsequently makes use of numerical simulations to discuss the squeezing of the mechanical and the optical output quadratures. Finally, we conclude with an outlook of possible future directions.

2. Dynamics equations

We consider an optomechanical resonator described by Fig. 1 of Ref. [15]. The dynamics equations of a mechanical oscillator coupled to a driven cavity is usually derived from a single-mode Hamiltonian [12, 23, 24, 27] as,

$$\ddot{x}_m + \Gamma_m \dot{x}_m + \Omega_m^2 x_m - \beta'' \Omega_m = g_M \Omega_m |\alpha(t)|^2 + \frac{F_{th}}{M x_{ZPF}}, \tag{1a}$$

$$\dot{\alpha} = \left[i(\Delta + \frac{g_M}{x_{ZPF}} x) - \frac{\kappa}{2} \right] \alpha(t) - i\varepsilon^{in} + \sqrt{\kappa}\alpha^{in}, \tag{1b}$$

x_m and p_m are the dimensionless position and momentum operators of the mechanical oscillator related to their counterparts operators of the nanobeam as $x = \sqrt{\frac{\hbar}{2M\Omega_m}} x_m = x_{ZPF} x_m$ and $p = \frac{\hbar}{x_{ZPF}} p_m$, with $[x_m, p_m] = 2i$. The parameters $g_M = \sqrt{2}\omega_c \frac{x_{ZPF}}{d_0}$, d_0, Ω_m and ε^{in} are respectively the optomechanical coupling, the cavity length, the mechanical frequency and the amplitude of the input laser beam. The cavity decay rate and the mechanical damping of the

mechanical oscillator are respectively represented by κ and Γ_m. The laser-cavity detuning is $\Delta = \omega_\ell - \omega_c$ with ω_c the optical cavity mode frequency and ω_ℓ the laser frequency. The terms F_{th} and α^{in} represent the Langevin force fluctuations and the input laser fluctuations. The terms $\beta'' = \frac{\beta' x_{ZPF}^2 x_m^3}{\Omega_m}$ and $g_M x_m \alpha$ represent the mechanical and optical anharmonic terms. When the nanoresonator is subjected to a large displacement amplitudes, it displays a striking nonlinearity β'' in its response. This comes about because the flexure causes the beam to lengthen, which at large amplitudes adds a significant correction to the overall elastic response of the beam [26]. The optical anharmonic term is another kind of Kerr medium, which has a mechanical origin: the radiation pressure induces a coupling between the position of the doubly-clamped flexural resonator and the phase-intensity of the light beam, thus modifying the optical path known as the phase shift.

One can derived from the set of equations Eqs.(1)the following nonlinear Quantum Langevin Equations (QLEs) [23]

$$\dot{x}_m = \Omega_m p_m \tag{2a}$$

$$\dot{p}_m = -\Omega_m x_m - \Gamma_m p_m + g_M \alpha^\dagger \alpha + \beta'' + F_{th} \tag{2b}$$

$$\dot{\alpha} = \left[i(\Delta + g_M x_m) - \frac{\kappa}{2} \right] \alpha - i\varepsilon^{in} + \sqrt{\kappa}\alpha^{in} \tag{2c}$$

$$\dot{\alpha}^\dagger = \left[-i(\Delta + g_M x_m) - \frac{\kappa}{2} \right] \alpha^\dagger + i\varepsilon^* + \sqrt{\kappa}\alpha^{in\dagger}. \tag{2d}$$

By setting the time derivatives to zero in the set of nonlinear Eqs. (2), the stationary values of the position of the oscillator and the amplitude of the cavity field are

$$\bar{x}_m = 2\frac{g_M}{\Omega_m}|\bar{\alpha}|^2, \quad |\bar{\alpha}|^2 = \frac{2\kappa P_{in}}{\hbar\omega_\ell \left((\Delta + g_M x_m)^2 + \frac{\kappa^2}{4} \right)}. \tag{3}$$

The values of $\bar{x}$ obey the following third order algebraic equation,

$$\bar{x}^3 + \frac{2\Delta x_{ZPF}}{g_M}\bar{x}^2 + (4\Delta^2 + \kappa^2)\frac{x_{ZPF}^2}{4g_M^2}\bar{x} - \frac{4\kappa x_{ZPF}^3 P_{in}}{\hbar\Omega_m\omega_\ell g_M} = 0. \tag{4}$$

From Eqs. (3) and (4), it appears that both $\bar{\alpha}$ and $\bar{x}_m$ increase when the input laser power P_{in} increases.

Using the experimental parameters of Ref. [15] at the detuning of $\Delta = \Omega_m$ and for $P_{in} = 1\,\text{mW}$, we obtain the following values of $\bar{x}$ which are in the range of those obtained experimentally in Refs. [14, 17]: 1.28×10^{-13}, $-1.09 \times 10^{-8} + 7.43 \times 10^{-10}i$, $-1.09 \times 10^{-8} - 7.43 \times 10^{-10}i$. The first solution, which is real and small, corresponds to the stable regime of the mechanical resonator, while the two conjugate others, which have the same module ($|\bar{x}| \approx 1.09 \times 10^{-8}$), correspond to the unstable regime.

For $|\bar{\alpha}| \gg 1$ (satisfied in Ref. [15]), the above QLEs can be linearized by expanding the operators around their steady states: $x_m = \bar{x}_m + \delta x_m$ and $\alpha = \bar{\alpha} + \delta\alpha$. By introducing the vector of quadrature fluctuations $u(t) = (\delta x_m(t), \delta p_m(t), \delta I(t), \delta\varphi(t))^T$ and the vector of noises $n(t) = (0, F_{th}(t), \sqrt{\kappa}\delta I^{in}(t), \sqrt{\kappa}\delta\varphi^{in}(t))^T$,

where $\delta I = (\delta\alpha^\dagger + \delta\alpha)$, $\delta\varphi = i(\delta\alpha^\dagger - \delta\alpha)$ are the intracavity quadratures of the intensity and the phase , and the corresponding hermitian input noise operators δI^{in} , $\delta\varphi^{in}$, the linearized dynamics of the system can be written in a compact form

$$\dot{u}(t) = Au(t) + n(t), \tag{5a}$$

with

$$A = \begin{pmatrix} 0 & \Omega_m & 0 & 0 \\ \Omega_m(\beta - 1) & -\Gamma_m & G & 0 \\ 0 & 0 & -\frac{\kappa}{2} & -\tilde{\Delta} \\ G & 0 & \tilde{\Delta} & -\frac{\kappa}{2} \end{pmatrix}, \tag{5b}$$

The higher order of fluctuations are safely neglected. The linearized QLEs show that the mechanical mode is coupled to the cavity mode quadrature fluctuations by the effective optomechanical coupling $G = g_M|\bar{\alpha}|$, which can be made large by increasing the input laser power P_{in}. $\beta = \frac{3\beta' x_{ZPF}^2 \bar{x}_m^2}{\Omega_m^2}$ and $\tilde{\Delta} = \Delta + g_M\bar{x}_m$ denote the dimensionless geometrical nonlinearity and the effective detuning. The range values of the geometrical and optical nonlinearities are given in the Table 1. One remarks that β and η increase when $\bar{x}_m$ increases and they reach their maximum values at the detuning $\Delta \approx \Omega_m$. As expected in the Table 1, the optical and the mechanical effects are respectively highly pronounced at the optical ($\Delta \approx 0$) and the mechanical ($\Delta \approx \Omega_m$) resonances [12]. This leads us to investigate the squeezing at this particular sidebands.

Detuning Δ	Mean displacement of the nanobeam $\bar{x}$(m)	Range of values of nonlinearities
0	2.77×10^{-11}	$\eta \in [2.54 \times 10^{-3}; 6.79 \times 10^{-2}]$
	7.42×10^{-10}	$\beta \in [7.87 \times 10^{-6}; 5.72 \times 10^{-4}]$
Ω_m	1.27×10^{-13}	$\eta \in [1.17 \times 10^{-5}; 1]$
	1.09×10^{-8}	$\beta \in [1.66 \times 10^{-10}; 1.22]$

Table 1: The range of values of the optical nonlinearity η and the geometrical nonlinearity β at the detuning $\Delta = 0$ and $\Delta = \Omega_m$ respectively, using the parameters of Ref.[15].

3. Squeezing of the mechanical quadratures

The dynamics of mechanical fluctuations is obtained by writing Eqs. (5) in the Fourier space,

$$B(\Omega)u(\Omega) + n(\Omega) = 0, \tag{6a}$$

where

$$B(\Omega) = \begin{pmatrix} i\Omega & \Omega_m & 0 & 0 \\ \Omega_m(\beta - 1) & (i\Omega - \Gamma_m) & G & 0 \\ 0 & 0 & (i\Omega - \frac{\kappa}{2}) & -\tilde{\Delta} \\ G & 0 & \tilde{\Delta} & (i\Omega - \frac{\kappa}{2}) \end{pmatrix}, \tag{6b}$$

Solving the matrix equation straightforwardly, we obtain the solution for the mechanical displacement operator to be

$$\begin{aligned}\chi_{eff}^{-1}(\Omega)\delta x_m(\Omega) = a_1 G\Omega_m\sqrt{\kappa}\left(\tilde{\Delta}^2+\frac{\kappa}{4}^2-\omega^2+i\kappa\Omega\right)\\ \times\left[-\tilde{\Delta}\delta\varphi^{in}+\left(-i\Omega+\frac{\kappa}{2}\right)\delta I^{in}\right]+\Omega_m F_{th},\end{aligned}\tag{7}$$

where

$$a_1=\left[\left(\tilde{\Delta}^2+\frac{\kappa}{4}^2-\Omega^2\right)^2+\kappa^2\Omega^2\right]^{-1},\tag{8}$$

and

$$\chi_{eff}(\Omega)=(\Omega_{eff}^2-\Omega^2-i\Omega\Gamma_{eff})^{-1},\tag{9}$$

is the effective susceptibility of the oscillator with the effective resonance frequency and damping rate given by

$$\Omega_{eff}^2(\Omega)=\Omega_m^2\left(1+a_1G^2\frac{\tilde{\Delta}}{\Omega_m}\left(\tilde{\Delta}^2+\frac{\kappa}{4}^2-\Omega^2\right)-\beta\right),\tag{10}$$

$$\Gamma_{eff}(\Omega)=\Gamma_m-a_1G^2\Omega_m\tilde{\Delta}\kappa.\tag{11}$$

By using the correlation functions of the noise sources for a coherent beam in the frequency domain, the oscillator position and the momentum variances are defined by,

$$\langle\delta x_m^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\Omega|\chi_{eff}|^2S_x,\tag{12}$$

$$\langle\delta p_m^2\rangle=\frac{1}{2\pi}\int_{-\infty}^{+\infty}d\Omega\frac{\Omega^2}{\Omega_m^2}|\chi_{eff}|^2S_x,\tag{13}$$

where the position noise spectrum is given by

$$\begin{aligned}S_x(\Omega)=a_1G^2\Omega_m^2\frac{\kappa}{2}\left(\tilde{\Delta}^2-\Omega^2+\frac{\kappa}{4}^2-i\kappa\tilde{\Delta}\right)\\ +2\Gamma_m\Omega_m\Omega\left(1+\coth\left(\frac{\hbar\Omega}{2k_BT}\right)\right).\end{aligned}\tag{14}$$

At the quasi resonant frequency ($\Omega\approx\Omega_m$) and for $\coth\left(\frac{\hbar\Omega}{2k_BT}\right)\approx\frac{2k_BT}{\hbar\Omega}$ (satisfied with experimental parameters used), the exact solutions of integrals (12) and (13) are given by

$$\langle\delta x_m^2\rangle=\frac{\Omega_m^2}{4\Gamma_{eff}\Omega_{eff}^2}a_2,\tag{15}$$

$$\langle\delta p_m^2\rangle=\frac{1}{4\Gamma_{eff}}a_2,\tag{16}$$

where

$$a_2 = \frac{\frac{G^2}{\Omega_m^2}\kappa\left(\frac{\tilde{\Delta}^2}{\Omega_m^2} + \frac{\kappa^2}{4\Omega_m^2} - 1 - i\frac{\kappa\tilde{\Delta}}{\Omega_m^2}\right)}{\left(\frac{\tilde{\Delta}^2}{\Omega_m^2} + \frac{\kappa^2}{4\Omega_m^2} - 1\right)^2 + \frac{\kappa^2}{\Omega_m^2}} + 4\Gamma_m\left(1 + \frac{2k_BT}{\hbar\Omega_m}\right). \tag{17}$$

These position and momentum variances should satisfy the Heisenberg relation,

$$\langle\delta x_m^2\rangle\langle\delta p_m^2\rangle \geq |\frac{1}{2}[x_m, p_m]|^2, \tag{18}$$

that is,

$$\langle\delta x_m^2\rangle\langle\delta p_m^2\rangle \geq 1. \tag{19}$$

There is no condition on the individual quadratures of relation (19). However, for a standard quantum limit (SQL) of 1, the coherent states must satisfy $\langle\delta x_m^2\rangle = \langle\delta p_m^2\rangle = 1$. When one variance is below the SQL, i.e., $\langle\delta x_m^2\rangle < 1$ or $\langle\delta p_m^2\rangle < 1$, the corresponding quadrature is said to be squeezed. Generally, the squeezed states are characterized by the asymmetry between its quadratures which is mostly introduced by the nonlinear effects. According to Eqs. (10) and (15), only $\langle\delta x_m^2\rangle$ depends on the geometrical nonlinearity through the term $\frac{\Omega_m^2}{\Omega_{eff}^2}$. Contrariwise, the value of $\langle\delta p_m^2\rangle$ is obtained by using experimental parameters of Ref. [15] in Eq. (16) and it appears to be squeezed up to about 37% (see Fig.1). Therefore, the position variance is deduced from the mean energy of the nanoresonator in the steady state,

$$E = \frac{\hbar\Omega_m}{4}(\langle\delta x_m^2\rangle + \langle\delta p_m^2\rangle) \equiv \hbar\Omega_m\left(n_{eff} + \frac{1}{2}\right), \tag{20}$$

by substituting the effective phonon number with the experimental value $n_{eff} = 0.85 \pm 0.08$ of Ref. [15]. The value obtained is $\langle\delta x_m^2\rangle \approx 4.44$ which is unsqueezed (see Fig. 2).

Indeed, the ratio $\frac{\Omega_m^2}{\Omega_{eff}^2}$ increases when β increases (for the high mechanical displacements) and rises up the position variance $\langle\delta x_m^2\rangle$. So, as the rethermalization time of the mechanical resonator or the decoherence time [16, 15], the geometrical nonlinearity limits the squeezing and some quantum effects [21]. In fact, β depends on the bending moment of the resonator and takes into account its internal vibrations. These internal vibrations increase for large bending and contribute to the re-thermalization of the resonator at the low temperatures. These effects are reversed to those of the Kerr nonlinearities which improve the squeezing [8]. For $\langle\delta x_m^2\rangle \approx 4.44$, β is evaluated to be about 0.88 (see Fig. 2) and is in the domain given in Table 1 and corresponds to the high mechanical displacements for the nanoresonators [17]. It also appears that for $\beta = 0.1$, the position variance $\langle\delta x_m^2\rangle$ is under the standard quantum limit (see Fig. 2), allowing therefore the squeezing (up to about 31%). In order to investigate the effect of the optical nonlinearity η on the mechanical squeezing, we consider $\eta \neq 0$ in the expression $\frac{\tilde{\Delta}}{\Omega_m} = \frac{\Delta}{\Omega_m} + \eta$ which appears in Eqs. (15) and (16). Figs. 3 shows the effect of η on the position variance $\langle\delta x_m^2\rangle$ for $\beta = 0.1$. One remarks

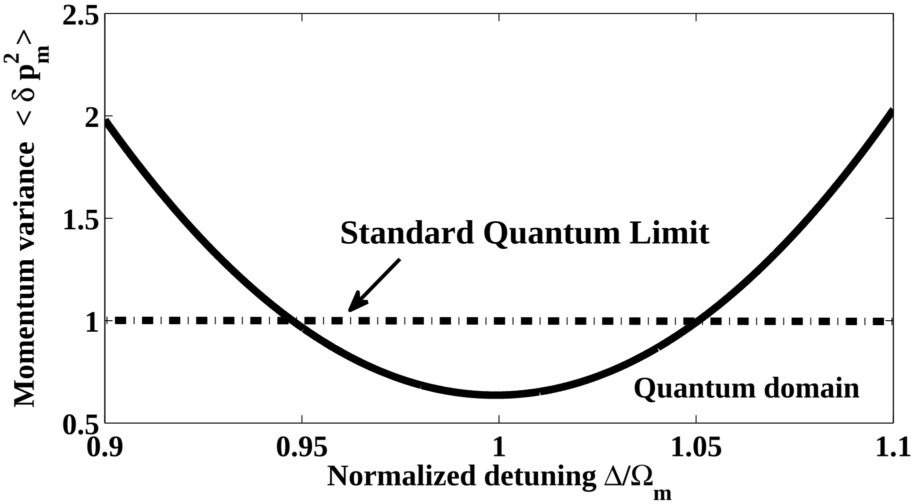

Figure 1: Plot of the momentum variance $\langle \delta p_m^2 \rangle$ versus normalized detuning $\frac{\Delta}{\Omega_m}$ for $\eta = 0$, using experimental parameters of Ref.[15]. The value of $\frac{\Delta}{\Omega_m} = 1$ corresponds to the momentum variance $\langle \delta p_m^2 \rangle = 0.6362$ which means that the momentum is squeezed up to about 37%.

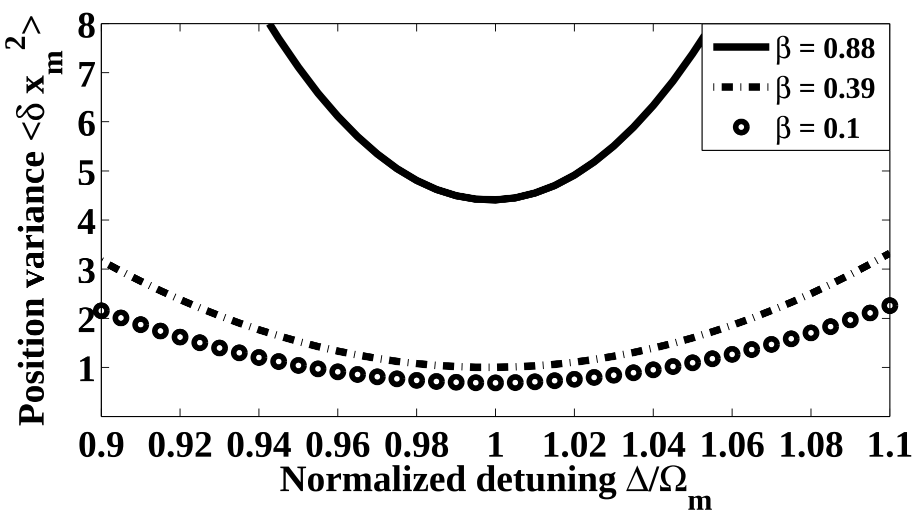

Figure 2: Plot of the position variance $\delta x_m^2 \rangle$ versus normalized detuning $\frac{\Delta}{\Omega_m}$ for different values of β with $\eta = 0$. The dot dashed line is plotted for $\beta = 0.39$ and corresponds to the Standard Quantum Limit ($\frac{\Delta}{\Omega_m} = 1$; $\langle \delta x_m^2 \rangle = 1$). The full line is plotted with $\beta = 0.88$ and experimental parameters of Ref.[15] and shows that the position is unsqueezed ($\frac{\Delta}{\Omega_m} = 1$; $\langle \delta x_m^2 \rangle \approx 4.44$). The cercled line is plotted for $\beta = 0.1$ and shows that the position is squeezed ($\frac{\Delta}{\Omega_m} = 1$; $\langle \delta x_m^2 \rangle \approx 0.69$).

in Fig. 3.a that the position variance $\langle \delta x_m^2 \rangle$ becomes unsqueezed for high values of η ($\eta > 0.042$). This effect of η is similar to that of the β shown in Fig. 2. It appears in Fig. 3.b that η shifts the optimal squeezing towards the left. Since η is always present in optomechanical systems, it is then important to quantify it in experiments in order to determine exactly at which detuning the optimal squeezing can be evaluated. On the Fig. 3.c where the two mentioned effects of η are represented, $\langle \delta x_m^2 \rangle$ increases with η and the optimal position squeezing is not always at $\frac{\Delta}{\Omega_m} = 1$ but depends on the value of η in the range $\frac{\Delta}{\Omega_m} \in [0.9; 1]$. One also notes that the effects of η on the momentum variance are the same as these described on Figs. 3.

On the other hand, the squeezing is improved despite such nonlinearities in the nanobeams when the temperature is tuned down and/or the system is carried in a regime of strong optomechanical coupling.

4. Squeezing of the optical output quadratures

From the matrix equation (6) we also obtain the solutions for the intracavity phase and intensity quadratures operators to be

$$\delta I = -a_3 \left[\tilde{\Delta} G \delta x_m - \tilde{\Delta} \sqrt{\kappa} \delta \varphi^{in} + \sqrt{\kappa} \left(-i\Omega + \frac{\kappa}{2} \right) \delta I^{in} \right], \tag{21}$$

and

$$\delta \varphi = a_3 \left[\left(G \delta x_m + \sqrt{\kappa} \delta \varphi^{in} \right) \left(-i\Omega + \frac{\kappa}{2} \right) + \tilde{\Delta} \sqrt{\kappa} \delta I^{in} \right], \tag{22}$$

with

$$a_3 = \left[\left(-i\Omega + \frac{\kappa}{2} \right)^2 + \tilde{\Delta}^2 \right]^{-1}. \tag{23}$$

In order to analyze their squeezing, we use the well-known input-output relation [28]

$$\alpha^{out} = -\alpha^{in} + \sqrt{\kappa} \alpha, \tag{24}$$

and then deduce

$$\delta I^{out} = -a_3 \left[\tilde{\Delta} \sqrt{\kappa} G \delta x_m - \tilde{\Delta} \kappa \delta \varphi^{in} + \left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2 \right) \delta I^{in} \right], \tag{25}$$

and

$$\delta \varphi^{out} = a_3 \left[\sqrt{\kappa} G \left(-i\Omega + \frac{\kappa}{2} \right) \delta x_m + \left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2 \right) \delta \varphi^{in} + \tilde{\Delta} \kappa \delta I^{in} \right]. \tag{26}$$

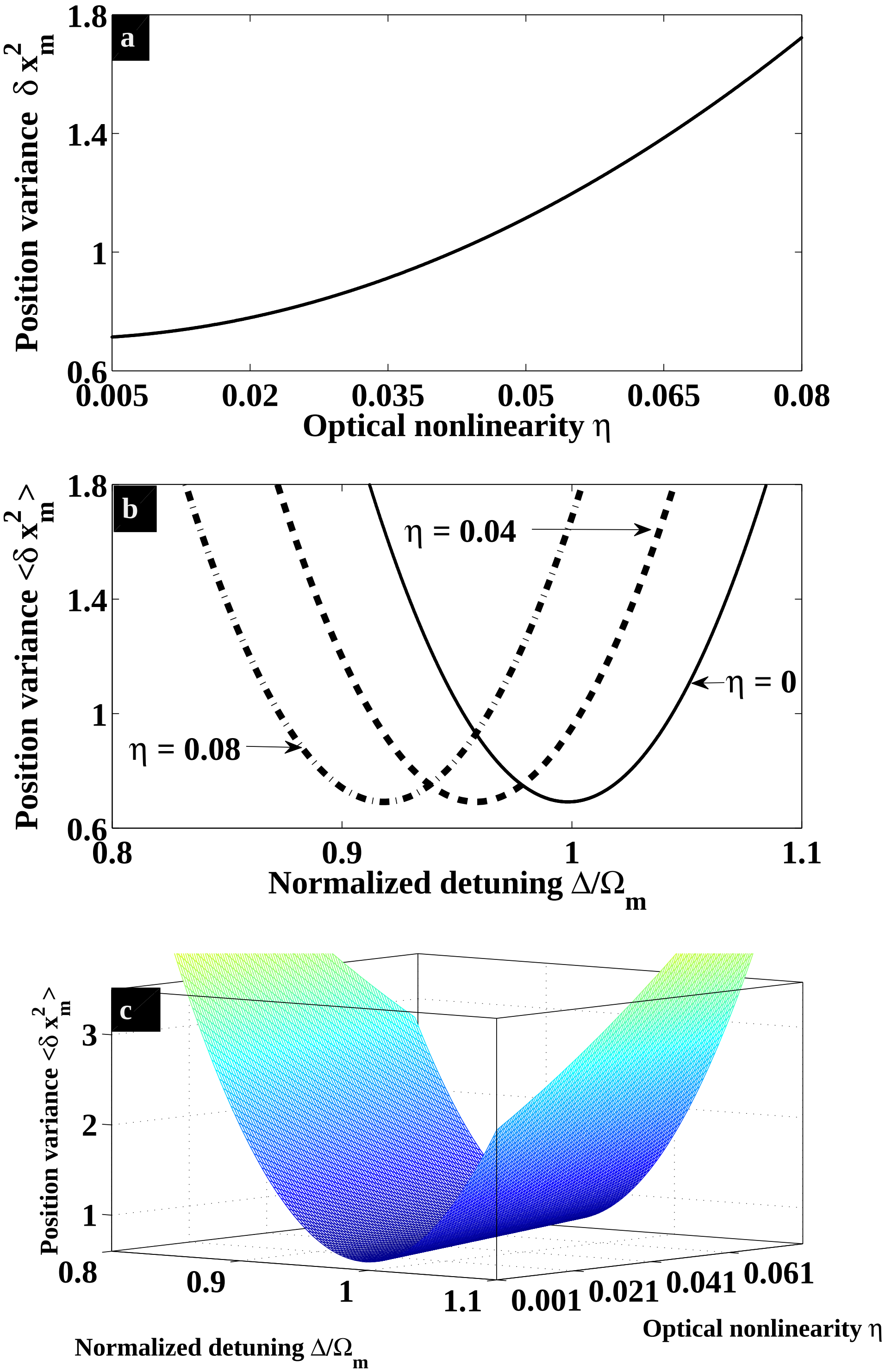

Figure 3: Effect of optical nonlinearity η on the position variance for $\beta = 0.1$. a. Position variance versus η shows that $\langle \delta x_m^2 \rangle$ becomes unsqueezed when η increases. b. shows that optimum squeezing shifts towards the left when η increases. c. shows the combined effects of η on $\langle \delta x_m^2 \rangle$.

By using the spectral density $S_{A^{out}}(\Omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega e^{-i(\Omega+\omega)t}\langle\delta A^{out}(\Omega)\delta A^{out}(\omega)\rangle$, we obtain the output spectrum of the intensity and phase as,

$$\begin{aligned} S_{I^{out}}(\Omega) &= a_1\tilde{\Delta}^2 G^2\kappa|\chi_{eff}|^2 S_x(\Omega) + \frac{a_1}{2}\left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right)^2 \\ &\quad + a_1\tilde{\Delta}^2\kappa^2 + A(\Omega) - B(\Omega) - C(\Omega), \end{aligned} \tag{27}$$

$$\begin{aligned} S_{\varphi^{out}}(\Omega) &= a_1 G^2\kappa\left(\frac{\kappa^2}{4} + \Omega^2\right)|\chi_{eff}|^2 S_x(\Omega) + \frac{a_1}{2}\left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right)^2 \\ &\quad + a_1\tilde{\Delta}^2\kappa^2 + G^2\kappa\Omega_m(D(\Omega) + E(\Omega)), \end{aligned} \tag{28}$$

where,

$$A(\Omega) = a\tilde{\Delta}G^2\Omega_m\kappa(\Omega_{eff}^2 - \Omega^2)(\tilde{\Delta} - \Omega), \tag{29}$$

$$B(\Omega) = b\tilde{\Delta}G^2\Omega_m\kappa\Omega\Gamma_{eff}(\tilde{\Delta} - \Omega), \tag{30}$$

$$C(\Omega) = \tilde{\Delta}G^2\frac{\kappa^2}{4}\Omega_m[2b(\Omega_{eff}^2 - \Omega^2) - 2a\Omega\Gamma_{eff}], \tag{31}$$

$$D(\Omega) = \left[(\Omega_{eff}^2 - \Omega^2)\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \Omega^2\right) + \kappa\Omega^2\Gamma_{eff}\right]\left(\frac{c}{2} - \Omega d\right)\kappa a_1, \tag{32}$$

$$E(\Omega) = \left[\kappa(\Omega_{eff}^2 - \Omega^2) - \Gamma_{eff}\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \Omega^2\right)\right]\left(\Omega c + \frac{\kappa^2}{2}d\right)\Omega a_1, \tag{33}$$

with

$$a = a_1^2\kappa\left[\tilde{\Delta}\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \Omega^2\right) - \Omega\left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right)\right], \tag{34}$$

$$b = a_1^2\left[\left(\tilde{\Delta}^2 + \frac{\kappa^2}{4} - \Omega^2\right)\left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right) + \tilde{\Delta}\kappa^2\Omega\right], \tag{35}$$

$$c = 2(\Omega - \tilde{\Delta})\left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right) + \kappa^2\tilde{\Delta} \tag{36}$$

$$d = 2\tilde{\Delta}(\Omega - \tilde{\Delta}) - \left(\Omega^2 + \frac{\kappa^2}{4} - \tilde{\Delta}^2\right). \tag{37}$$

In Eqs. (27) and (28), the first terms are proportional to the position spectrum S_x (Eq. (14)) and to the effective mechanical susceptibility (Eq. (9)). These terms derive from the mechanical fluctuations of the oscillator. The other terms in (27) and (28) originate to the fluctuations of the input beam. Assuming that the system is in the quasi resonant regime ($\Omega \approx \Omega_m$), all the contributions related to the input beam fluctuations take constant values. The expressions (27) and (28) can now be integrated. By using the residues theorem and the Cauchy-Goursat theorem, one readily obtains

$$\langle\delta I^{out\,2}\rangle = \frac{\frac{\tilde{\Delta}^2}{\Omega_m^2}\frac{G^2}{\Omega_m^2}\kappa}{\left(\frac{\tilde{\Delta}^2}{\Omega_m^2} + \frac{\kappa^2}{4\Omega_m^2} - 1\right)^2 + \frac{\kappa^2}{\Omega_m^2}}\langle\delta x_m^2\rangle, \tag{38}$$

and

$$\langle \delta\varphi^{out\,2}\rangle = \frac{\left(1+\frac{\kappa^2}{4\Omega_m^2}\right)\frac{G^2}{\Omega_m^2}\kappa}{\left(\frac{\tilde{\Delta}^2}{\Omega_m^2}+\frac{\kappa^2}{4\Omega a_m^2}-1\right)^2+\frac{\kappa^2}{\Omega_m^2}}\langle \delta x_m^2\rangle, \tag{39}$$

where $\langle \delta x_m^2\rangle$ is given by Eq. (15).

At the mechanical resonance ($\tilde{\Delta} \approx \Omega_m$) where it is established above that the position variance is unsqueezed ($\langle \delta x_m^2\rangle = 4.44$), we deduce from Eqs. (38) and (39) that both optical variances are unsqueezed. However, the effective detuning $\tilde{\Delta} \approx \Delta + g_M \bar{x}_m$ leads at the optical resonance ($\Delta = 0$) to

$$\frac{\tilde{\Delta}}{\Omega_m} \approx \frac{g_M \bar{x}_m}{\Omega_m} = \eta. \tag{40}$$

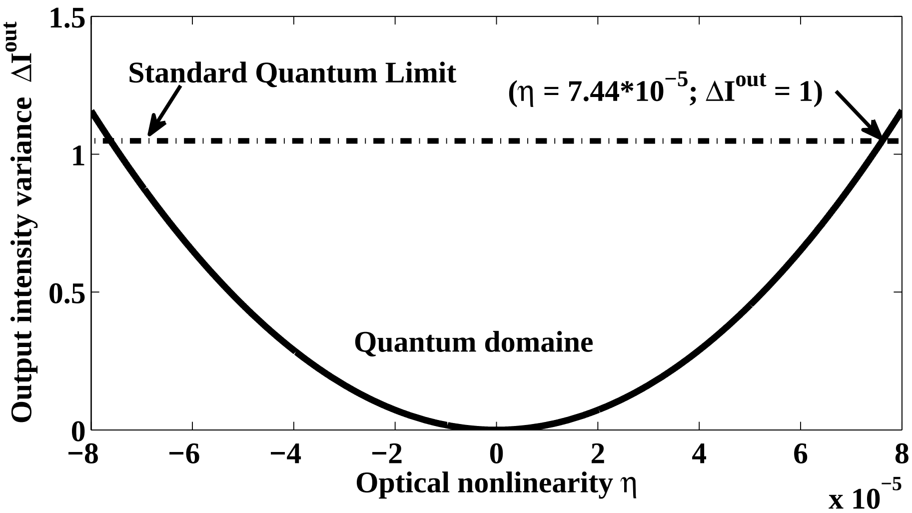

Figure 4: Plot of the mean square fluctuation of the output light ΔI^{out} versus the optical nonlinearity η for $\beta = 5.72\times10^{-4}$. With the low input power used for the system $P_{in} \leq 30\,\mu$W [16], the output intensity is squeezed ($|\eta| \leq 7.44 \times 10^{-5}$).

Fig. 4 shows that ΔI^{out} is squeezed when $|\eta| \leq 7.44 \times 10^{-5}$ which is obtained with the input power $P_{in} \leq 30\,\mu$W [16]. This means that at the optical resonance and for the low input power, the optomechanical cavity behaves as an optical filter or *noise eater* [10]. Thus, the quantum fluctuations (shot noise) in the input coherent laser beam ($\Delta I^{in} = 1$) are reduced after been reflected out of the optomechanical cavity ($\Delta I^{out} < 1$). To improve this squeezing at the optical resonance one can increase η (for the negative values of the detuning) or decrease η (for the positive values of the detuning) (see Fig. 4). By using

Eqs.(2c) and (24), one can expressed output field as

$$\alpha^{out} = \frac{a_4 + i a_5}{\left(\frac{\Delta}{\Omega_m} + \eta - 1\right)^2 + \frac{\kappa^2}{4\Omega_m^2}} \tag{41}$$

with

$$a_4 = \frac{\kappa^2}{4\Omega_m^2}\alpha^{in} + \left(\frac{\Delta}{\Omega_m} + \eta - 1\right)\left(\left(\frac{\Delta}{\Omega_m} + \eta - 1\right)\alpha^{in} + \frac{\sqrt{\kappa}}{\Omega_m}\varepsilon^{in}\right), \tag{42}$$

$$a_5 = \left(\left(\frac{\Delta}{\Omega_m} + \eta - 1\right)\frac{\kappa}{\Omega_m}\alpha^{in} - \frac{\kappa\sqrt{\kappa}}{2\Omega_m^2}\varepsilon^{in}\right), \tag{43}$$

where $\varepsilon^{in} = \sqrt{\frac{2\kappa P_{in}}{\hbar\Omega_m}}$. One remarks that α^{out} depends only on the optical nonlinearity which allows us to quantify his effect on the output field. Fig. 5.a shows that the output field decrease when η increases. This means that it is important to control the value of η in order to obtain the output field needed. This control of η also gives the value of the detuning at which the output field is optimal (see Fig. 5.b). Fig. 5.c shows the two mentioned effects of η on the output field α^{out}.

It is also found that, as for the position, the decrease of temperature induces an improvement of intensity squeezing. Regarding the geometrical nonlinearity, it contributes to reduce the squeezing when it becomes large. But, it is generally weak around the optical resonance (see Table 1), so its effects are neglected at this sideband.

Let us recall that the reduction of the nonlinearities in the optomechanical systems, constitutes the key of many future quantum optomechanical applications. This requires the reduction of the mean displacement $\bar{x}$ of the nanoresonator or the increase of the fundamental mechanical frequency Ω_m. This consists to use a very high finesse cavity which can be excited by the low input power [16].

It should be noted that squeezing of nonlinear optomechanical systems in which an optical cavity mode is coupled quadratically rather than linearly to the position of mechanical oscillator have been studied in Ref. [29]. While Sete and Eleuch, in Ref. [22], investigating nonlinear effects in an optomechanical system containing a quantum well, have found that as a result of the nonlinearity induced by the optomechanical coupling, the transmitted field exhibits strong squeezing at certain hybrid resonance frequencies and system parameters.

5. Conclusion

We have presented an analytical study of the geometrical and optical nonlinear effects on the optomechanical squeezing. Contrary to the Kerr nonlinearities, it is found that these two nonlinearities reduced the squeezing. At the detuning $\tilde{\Delta} \approx \Omega_m$ where the displacement is important, the momentum is squeezed while the position squeezing is very restricted for the small values of β. This effect is

justified by the geometrical nonlinearity which depends on the bending moment of the resonator and takes into account its internal vibrations. At the optical resonance, the output intensity is squeezed when $|\eta|$ is small ($|\eta| < 7.44 \times 10^{-5}$).

In a future work, we will investigate the squeezing as a function of the scanning frequency (Ω_m) in order to study the squeezing at different resonances.

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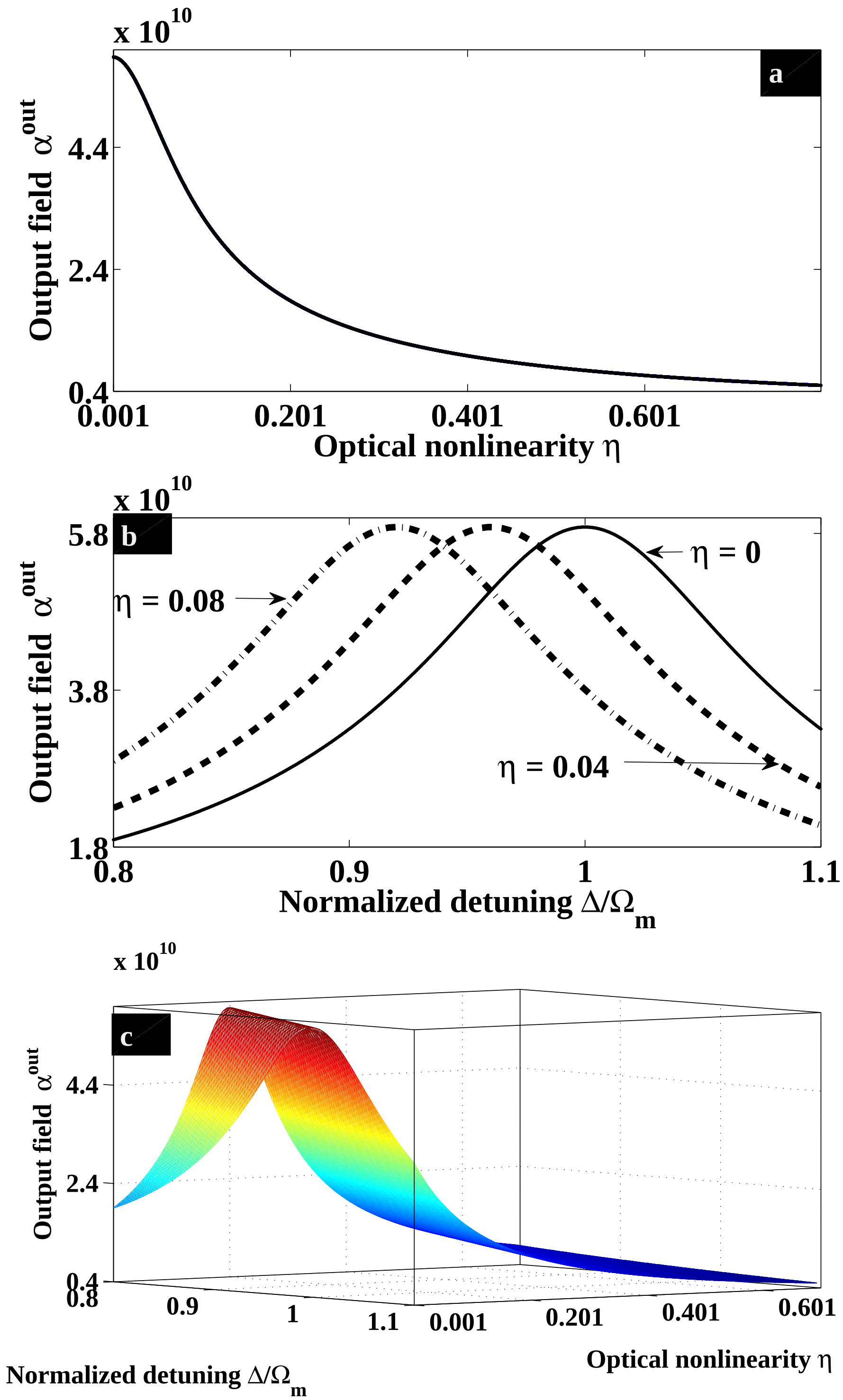

Figure 5: Effect of optical nonlinearity η on the output field. a. Output field versus η shows that α^{out} decreases when η increases. b. The optimal output field shifts towards the left when η increases. c. Combined effects of η on α^{out}.